Two detectors, a and b, are used such that a is a distance x from the wall and b is a distance x+d from the wall. The distance between the sensors is d

Assume that the intensity of light seen by each detector is an inverse funcion of the distance to the wall. While that would classically be an inverse square relationship, experiments show that it is not really a square law. For calculations, the actual power is denoted by n. A typical observed value for n might be 1.4

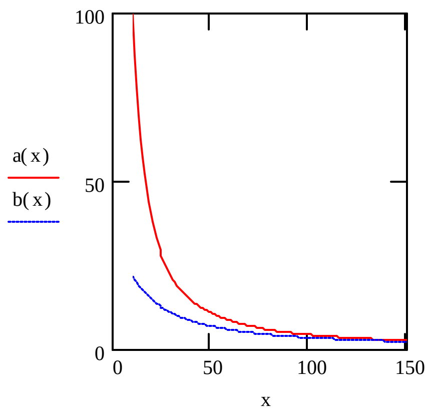
We also need some value for the reflectivity of the wall, k. Thus:

Wall reflectivity is $k := 2500$

Wall distance is in the range $x := 10 .. 150$ $d := 20$

Intensity power dropoff $n := 1.4$

and $a(x) := \frac{k}{x^n}$ $b(x) := \frac{k}{(x+d)^n}$



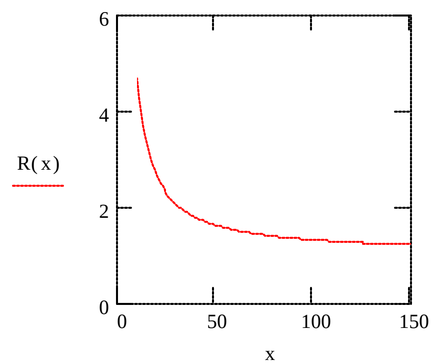
We can measure these intensities and calculate their ratio, R

$$R(x) := \frac{a(x)}{b(x)} \rightarrow \frac{1}{x^{1.4}} \cdot (x+20)^{1.4}$$

$$R(x) := \frac{(x+d)^n}{x^n}$$

The reflectivity, k, is now absent from the solution. A graph of R(x) as a function of x looks like this.

While we have now eliminated k as a variable, the result is not much better in terms of accuracy. At long range, the value of R is likely to be masked by noise. Furthermore, the ratio is quite small and the range of values of R is restricted which might make noise a significant problem.

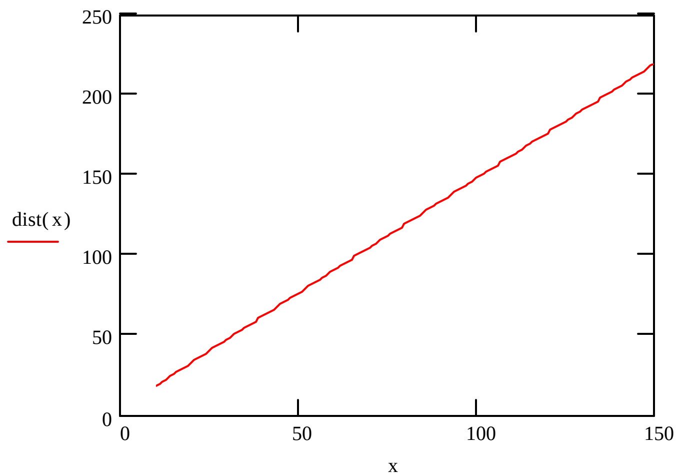


Assuming we can calculate R to a reasonable degree of accuracy, we can rearrange the expression above to get x in terms of R:

$$n := 2$$

$$\text{dist}(x) := \frac{d}{\sqrt[n]{R(x)} - 1}$$

If the value of n were truly 2, giving us an inverse square law, the solution to this equation is a direct, straight line graph which would tell us the value of x very neatly.



Notice that, to get a 'correct' answer, we need to know the distance between the sensors. This is not a significant issue since it just represents a scaling factor which can be used to give a convenient output scale for a particular configuration.

Now n is not 2. Observations suggest it actually has a value of about 1.4 and I would expect the value to change depending on the nature of the physical implementation.

At this point, I start to get worried. As it stands, the expression already needs three divisions and a square root, all in floating point. This is expensive computationally and I only have so much processor power to spare as I approach Mach 1 down the straights. Worse still, I would need to calculate fractional roots and I would probably have to reach with feverish hands for text books about Taylor series or some such horror.

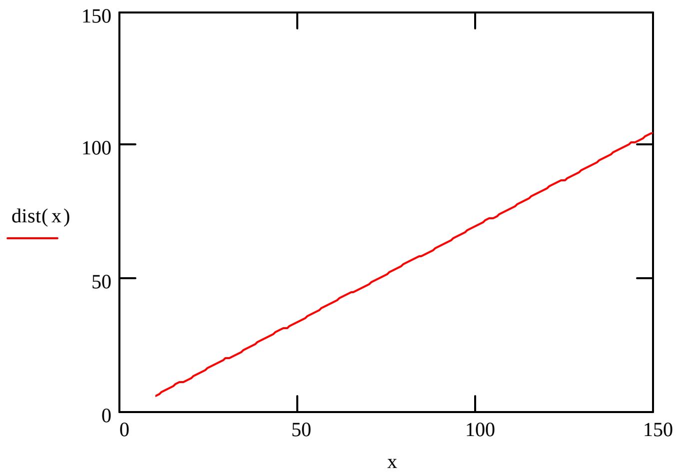
So, being lazy, I decide to see what would happen if I just pretended I did not need to do the square root. The expression for distance would then become:

Again we have

$$n := 1.4$$

$$R(x) := \frac{(x+d)^n}{x^n} \qquad \text{dist}(x) := \frac{d}{R(x) - 1}$$

And if I plot dist(x) as a function of x, I get:



Waddayaknow - it is still a straight line function. In fact for any reasonable value of n from 0.5 to 2.5, you get a perfectly usable straight line graph.

Now we are getting somewhere. I have only three divisions and a subtraction to perform. What happens if we go back to the original premise again? Remember that

$d := 10 \qquad n := 1.4$

$$a(x) := \frac{k}{x^n}$$

$$b(x) := \frac{k}{(x+d)^n}$$

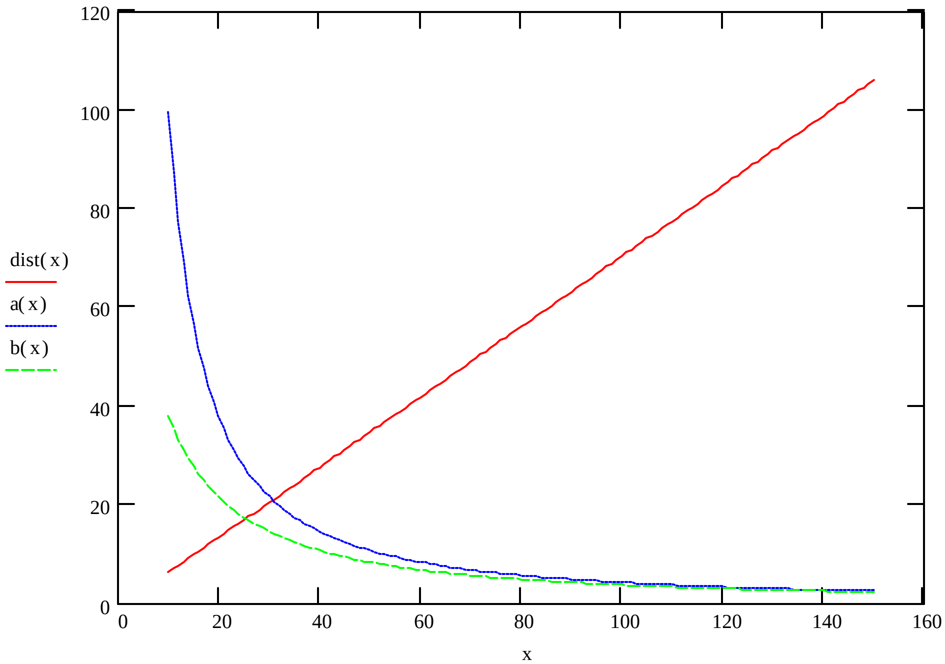
$$R(x) := \frac{a(x)}{b(x)}$$

$$dist(x) := \frac{d}{R(x) - 1}$$

$$dist(x) := \frac{d}{\left(\frac{a(x)}{b(x)} - 1\right)}$$

$$dist(x) := \frac{d \cdot b(x)}{a(x) - b(x)}$$

So lets have a graph because they are *so* comforting:



Now I must be honest and say that this looks like one of those awful trick calculations they used to give you at school. The kind where a a couple of pages of maths proved that 0 = 1 because of some dreadful assumption that fools like me could never spot.

In this case it will be something to do with not bothering with the root but that is a bit of a means to an end

Still, assuming I have not committed some other mathematical sin, we can use differential sensors and, with only a multiplication, a division and a subtraction, obtain a value for distance.

As a further caution however, you should note that it is in the very nature of these sensors that the signal to noise drops off significantly with distance. This is not really a problem so long as you dont imagine you can actually measure distance as opposed to simply knowing where you are well enough to avoid a collision.